

Dissipative dynamics of quantum discord under quantum chaotic environment

Y. Y. Xu^{1,2}, W. L. Yang¹ and M. Feng^{1*}

¹*State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China and*

²*Graduate School of the Chinese Academy of Sciences, Beijing 100039, China*

We investigate the dissipative dynamics of quantum discord in a decoherence model with two initially entangled qubits in addition to a quantum kicked top. The two qubits are uncoupled during the period of our study and one of them interacts with the quantum kicked top. We find that the long time behavior of quantum discord could be well described by the fidelity decay of the quantum kicked top; for short time behavior, however, the phase of the amplitude of the fidelity decay is necessary to provide more specific information about the system. We have made comparison between the quantum kicked top and multi-mode oscillator system in describing environment, and also compared the dynamics of the entanglement with that of quantum discord.

PACS numbers: 03.65.Ud, 05.45.Mt, 03.65.Yz, 03.67.Mn

It was a surprise that some quantum computing approaches could be carried out efficiently in the absence of entanglement [1]. This implies that entanglement does not involve complete quantum correlation and thereby a new terminology 'quantum discord' (QD) has drawn more and more attention [2–5] recently and has been considered to replace entanglement as a vital quantum resource for quantum information processing.

Since no quantum system is isolated from the environment, it is of fundamental and practical importance to study the dissipative dynamics of the QD. Modeling the environment by a multi-mode oscillator system [6] had been widely adopted, but recent research pointed out that the environment could also be simulated by the quantum chaos model [7], such as a quantum kicked top (QKT) [8]. It was shown that the quantum chaos model could simplify the treatment about environment compared with the multi-mode oscillator system model, and also efficiently simulate both Markovian and non-Markovian environments [9].

Our focus in the present work is on the dynamics of the QD under a quantum chaos model, in which the decoherence is from a QKT [10], and the decoherence could be reflected by fidelity decay (FD). We consider two qubits with one of them coupled to a QKT. The two qubits are initially entangled, but with no coupling during the period of our investigation. Our purpose is to investigate the physics behind the dynamics of the QD under this QKT-induced decoherence. To this end, we will check the dynamics of the QD with respect to the chaotic and regular regimes of the QKT. We find that FD is suitable for describing the long-term dynamics of the QD, which shows difference between chaotic and regular QKTs. Moreover, from the phase of the amplitude of FD (PAFD), we may discover more differences in the environment-induced behavior between chaotic and regular QKTs. Furthermore, we will try to explore the simi-

larity, from the dynamics of the QD, between the chaotic (regular) QKTs and Markovian (non-Markovian) multi-mode oscillator environment. In addition, it has been shown that the relative entropy entanglement (REE) is closely related to the QD [11]. As a result, a comparison of the QD with the REE would make us understand the dissipative dynamics from another angle.

Let us first briefly review the main points of QD. The QD is defined to quantify the difference of mutual information (MI) between the classical and quantum forms. In classical information theory, the correlation between two random variables A and B is described by MI using following two expressions: $I(A, B) = H(A) + H(B) - H(A, B)$ and $J(A, B) = H(A) - H(A|B)$, where $H(A)$ and $H(B)$ are the Shannon entropies, $H(A, B)$ is the joint Shannon entropy, and $H(A|B)$ is the conditional entropy introduced for quantifying the ignorance about the value of A given a known B . The two expressions above are equivalent in the classical case using Bayes' rule, and their quantum version is the replacement of the Shannon entropy by von Neumann entropy, i.e., $I(\rho_A, \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{A,B})$, and $J(\rho_A, \rho_B) = S(\rho_A) - S(\rho_A|\rho_B)$, where quantum conditional entropy $S(\rho_A|\rho_B)$ depends on the choice of measurement. If we perform a set of projectors $\{\Pi_j^B\}$ on the subsystem B , then the post-measurement states of the subsystem A with outcomes j is $\rho_{A|\Pi_j^B} = p_j \text{Tr}[\Pi_j^B \rho_{AB} \Pi_j^B]$ with $p_j = \text{Tr}[I \otimes \Pi_j^B \rho_{AB}]$, and the quantum conditional entropy is defined as $S(\rho_A|\rho_B) = \sum p_j S(\rho_{A|\Pi_j^B})$. So different choices of $\{\Pi_j^B\}$ result in different $S(\rho_A|\rho_B)$, and the QD is defined as the minimum difference of the quantum MI between $I(\rho_A, \rho_B)$ and $J(\rho_A, \rho_B)$, i.e., $Q = \min[I(\rho_A, \rho_B) - J(\rho_A, \rho_B)]$, where $J(\rho_A, \rho_B)$ is also called classical correlation (CC) [12].

Consider the QKT with following Hamiltonian [10]

$$H_E = \frac{\nu}{T} J_x + \frac{\eta}{2J} J_z^2 \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad (1)$$

where J_x and J_z are angular momentum components with $\hat{J} = J_x \hat{i} + J_y \hat{j} + J_z \hat{k}$, T is the time period, and

*Corresponding author: mangfeng@wipm.ac.cn

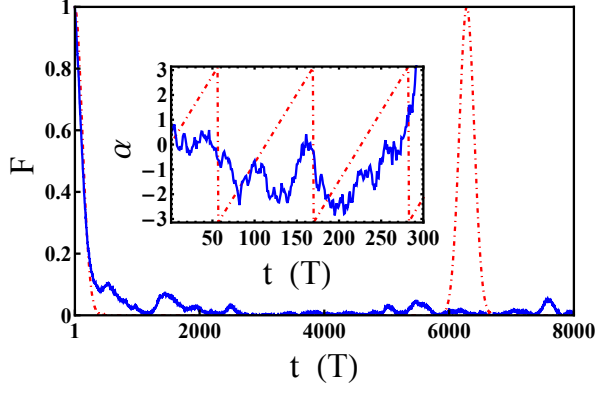


FIG. 1: (color online) Time evolution of the FD for chaotic (blue solid line) and regular (red dot-dashed line) QKTs in units of T , where the parameters are $J = 100$, $\nu = \pi/2$ and $\epsilon = 0.001$. We set $\eta = 20$ and 0.1 , respectively, for chaotic and regular QKTs. The inset shows the time evolution of the PAFD α for chaotic (blue solid line) and regular (red dot-dashed line) QKTs in short time scale, where, since the variation in the regular case is much larger than that in the chaotic case, we enlarge α by 40 times in the chaotic case, for a comparison with the regular case.

ν and η represent the precessional angle and the kicking strength, respectively. We assume that both the dynamics of the qubits and dissipations are much slower than dephasing. So we may only focus on the dephasing effect. We consider one of the qubits (A) is free and the other (B) interacts with QKT via the Hamiltonian

$$H = \epsilon \sigma_z^B V_E + H_E \quad (2)$$

where ϵ is the strength of the interaction between the qubit and the QKT, and σ_z^B is the Pauli operator of the qubit B . V_E is an operator of the QKT, which, for convenience, could be chosen as J_x in the following calculation. The corresponding Floquet operator can be constructed as

$$U = e^{-i(\nu + \epsilon \sigma_z^B) J_x} e^{-i \frac{\eta}{2J} J_z^2}. \quad (3)$$

In the classical limit $J \rightarrow +\infty$, the chaoticity degree is determined by the kicking strength η and the precessional angle ν . For example, for $\nu = \pi/2$, the classical kicked top is dominated by a regular motion for $\eta \leq 2.5$, whereas the motion is prevailed by chaos for $\eta \geq 3$ [13]. The regime with $2.5 \leq \eta \leq 3$ is for mixture of chaos and regularity. For our purpose in this work, we will only consider the regular and chaotic regimes, instead of the mixture regime.

Consider following initial state of the system,

$$\rho_{ABE} = \frac{1}{4} (I + \sum_{j=x,y,z} c_j \sigma_j^A \otimes \sigma_j^B) \otimes |\theta, \phi\rangle \langle \theta, \phi|, \quad (4)$$

where c_j are real numbers, and σ_j^A (σ_j^B) is the usual Pauli matrix acting on the subsystem A (B) [2]. The

initial QKT state $|\theta, \phi\rangle$ is a spin coherent state defined as [14]

$$|\theta, \phi\rangle = \exp[-i\theta(J_x \sin \phi - J_y \cos \phi)] |J, J\rangle, \quad (5)$$

where $|J, J\rangle$ is the eigenstate of \hat{J}^2 and J_z with eigenvalues $J(J+1)$ and J , respectively. The time evolution of the density matrix after n kicks can be described as

$$\rho'_{ABE} = U^n \rho_{ABE} (U^\dagger)^n, \quad (6)$$

with the corresponding reduced density matrix of the two qubits, in the basis $\{|1\rangle = |0\rangle_A |0\rangle_B, |2\rangle = |0\rangle_A |1\rangle_B, |3\rangle = |1\rangle_A |0\rangle_B, |4\rangle = |1\rangle_A |1\rangle_B\}$, given by

$$\rho'_{AB} = \begin{pmatrix} \frac{1+c_z}{4} & 0 & 0 & \frac{(c_x-c_y)f}{4} \\ 0 & \frac{1-c_z}{4} & \frac{(c_x+c_y)f}{4} & 0 \\ 0 & \frac{(c_x+c_y)f^*}{4} & \frac{1-c_z}{4} & 0 \\ \frac{(c_x-c_y)f^*}{4} & 0 & 0 & \frac{1+c_z}{4} \end{pmatrix}, \quad (7)$$

where $f = \langle \Psi'(t) | \Psi(t) \rangle = \langle \theta, \phi | \exp[i(\epsilon V_E + H_E)t] \exp[-i(-\epsilon V_E + H_E)t] | \theta, \phi \rangle$ is the amplitude of FD of the QKT [15], which is generally complex, i.e., $f = \sqrt{F} e^{i\alpha}$, where $F = |f|^2$ is the FD, and the PAFD α reflects the relative phase between the QKT states $|\Psi(t)\rangle$ and $|\Psi'(t)\rangle$. For a fully chaotic quantum system realized through adjusting the parameter η , FD behaves with Gaussian or exponential decay depending on the values of the perturbation strength [15]. Change of the parameter η moves the quantum system to the regular regime of the QKT, and then yields only the Gaussian decay and revivals of the FD [15, 16]. The corresponding revival time is $\tau = k/\epsilon$ with k the constant. Fig. 1 shows the FD for chaotic and regular QKTs. In our calculation here and following, the time unit is the period T , and the initial state $|\theta, \phi\rangle$ is a randomly chosen spin coherent state. As shown in Fig. 1, the fluctuation in the chaotic case could be understood as being from the finite numbers of the eigenstates of the Floquet operator U involved in the initial QKT state $|\theta, \phi\rangle$ [17], and the revivals in the regular case is related to the underlying periodic classical motion [16]. Both the fluctuation and the revival could be regarded as the memory effects. The inset in Fig. 1 shows the regular and chaotic behaviors reflected by PAFD: Their variations are regular and irregular respectively, and the amplitude of the variation in the regular case is much larger than that in the chaotic case.

The reduced density matrix of the two qubits in Eq. (7) is an X state, whose QD and CC could be calculated analytically, using the methods in [18], as

$$Q(\rho'_{A,B}) = 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - C(\rho'_{A,B}), \quad (8)$$

and

$$C(\rho'_{A,B}) = 1 - \min\left[-\sum_{k=0}^1 \frac{1 + (-1)^k \theta_1}{2} \log_2 \frac{1 + (-1)^k \theta_1}{2}, -\sum_{k=0}^1 \frac{1 + (-1)^k \theta_2}{2} \log_2 \frac{1 + (-1)^k \theta_2}{2}\right], \quad (9)$$

where $\lambda_1 = [1 + c_z + |(c_x - c_y)|\sqrt{F}]/4$, $\lambda_2 = [1 + c_z - |(c_x - c_y)|\sqrt{F}]/4$, $\lambda_3 = [1 - c_z + |(c_x + c_y)|\sqrt{F}]/4$, $\lambda_4 = [1 - c_z - |(c_x + c_y)|\sqrt{F}]/4$, $\theta_1 = |c_z|$, and $\theta_2 = \sqrt{[2(c_x^2 + c_y^2) + 2|c_x^2 - c_y^2|(\cos 2\alpha + |\sin 2\alpha|)]F/2}$. For a real or imaginary amplitude of the FD, θ_2 could be reduced to $\theta_2 = \sqrt{F} \max[|c_x|, |c_y|]$, which is similar to the result of the phase damping channel in [2], describing the dephasing induced by the Markovian multi-mode oscillator environment.

To be more clarified, we compare the QD with the REE for entanglement [19]. Direct calculation yields the REE in our case,

$$E = 1 + \beta' \log_2 \beta' + (1 - \beta') \log_2 (1 - \beta'), \quad (10)$$

where $\beta' = \max[0.5, \lambda_{\max}]$, with $\lambda_{\max} = \max[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$. We plot the evolution of QD and REE, along with CC and FD, in Figs. 2 and 3 for a comparison. In the chaotic regime, both the QD and REE are only determined by the FD, but in the regular regime, the REE still only depends on the FD, whereas QD behaves complicated: Its long time behavior is fully determined by FD; the short time behavior is more relevant to the PAFD α .

To further check the Markovian and non-Markovian dephasing effects of the QKT, we consider below the correspondence associated with the chaotic and regular QKTs, respectively. For the chaotic QKT, the PAFD α is nearly zero during the short time evolution of the FD. So the effect from the chaotic QKT is similar to the phase damping channel, a typical Markovian environment. Like the model of the phase damping channel, there are three kinds of CC dynamics in our case here, depending on the parameters c_j , i.e., the constant CC, CC with a sudden change, and CC with monotonic decay [2]. Our interest is in the case with a sudden change, where the QD can be maintained for some time [20]. To show similarities and differences between our decoherence model and the phase damping channel, the dynamics of QD, CC and others, whose short time behaviors are very similar to those under the phase damping channel [20], are plotted in Fig. 2. But for long time behavior, the situation is different: For example, there are fluctuations in the evolution of the QD for a finite angular momentum J , which does not happen in the phase damping channel. In addition, we could find from the Fig. 2 (a) the consistency in the short time evolution between the CC and the FD, and also between the REE and the FD. Furthermore, in comparison to the sudden death in the time evolution of REE, there is only instantaneous disappearance of the QD at some time points as shown in Fig. 2 (b), where

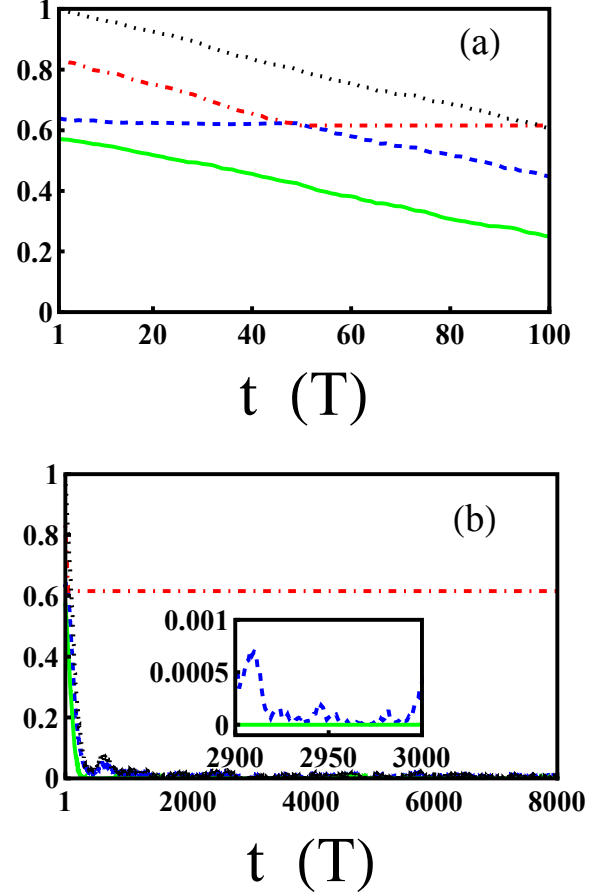


FIG. 2: (color online)(a) Short time evolution of the QD (blue dashed line), CC (red dash dotted line), REE (green solid line), and FD (black dotted line) in units of T with respect to the chaotic QKT, where the parameters of the FD are the same as in Fig. 1 and the initial state is with the parameters $c_x = 0.95$, $c_y = -0.85$, and $c_z = 0.85$. (b) Long time evolution of the QD, CC, REE, and FD under the same parameters in (a). The inset is a magnification for the time period $2900 < t/T < 3000$.

the small fluctuations of the QD are originated from the memory effects of the QKT. Those fluctuations disappear only in the classical limit, i.e., $J \rightarrow +\infty$ in our case, which corresponds strictly to the Markovian regime.

The memory effects of the QKT could be more evident in the treatment of the regular QKT, which resembles the dynamics in the non-Markovian environment [5]. From short time behaviors, we may find that before the CC becomes a constant, there exist oscillations in both the

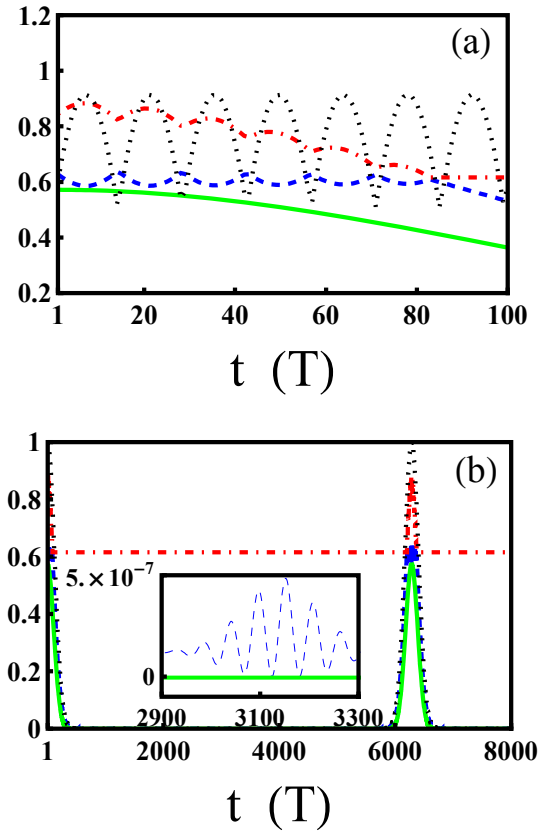


FIG. 3: (color online) (a) Short time evolution of the QD (blue dashed line), CC (red dash dotted line), REE (green solid line), and the compressed quantity $|\cos 2\alpha| + |\sin 2\alpha|$ (black dotted line) relevant to the regular QKT in units of T , where other parameters are the same as in Fig. 2 except $\eta = 0.1$. (b) Long time evolution of the QD (red solid line), CC (blue dashed line), REE (green solid line), and the quantity FD (black dotted line) under the same parameters in (a). The inset is a magnification for the time period $2900 < t/T < 3300$.

CC and the QD, as shown in Fig. 3 (a). Although these oscillations are very similar to those in a non-Markovian environment [4], their origins are different. The oscillation in the non-Markovian environment is related to the dephasing [21], whereas in our case it comes from the PAFD. In contrast, there is no oscillation in the short time behavior of the REE (see Fig. 3(a)), which could be considered as an important difference between entangle-

ment and QD. Another difference between entanglement and QD could be found in the long time evolution (See Fig. 3(b)), where REE experiences sudden death and birth, but not for QD.

Therefore, only the chaotic QKT in the classical limit describes the Markovian environment. The rest of the QKT, such as the finite J of the chaotic QKT as well as the regular QKT, corresponds to the non-Markovian environment. In this sense, the results in [20] are only relevant to the chaotic QKT in classical limit. It would be of great interests to revisit the problems in [20] by regular QKT or finite J of chaotic QKT. Moreover, entanglement is usually described by concurrence. If concurrence is employed in our model, however, the main results obtained above would be remained, because both REE and concurrence involve no PAFD. So entanglement could be distinguished from QD by PAFD regarding the regular regime of the environment.

Our model could be straightforwardly extended to some more complicated cases, such as two qubits interacting with a QKT commonly or with two QKTs independently, particularly from the initial state Eq. (4). As a result, our model would be useful to study either two locally located qubits experiencing the same environment or two distant qubits suffering different decoherence. Moreover, our model involves only dephasing induced by the QKT. It might be more interesting to take into account a model involving more complicated dissipation in the future.

In summary, we have studied the dissipative dynamics of the QD for two qubits initially entangled, but only one of them coupled by a QKT. We have checked the dynamics of the QD in both the chaotic and regular regimes of the QKT, in comparison to entanglement. A comparison of the dynamics of the QD under QKT with the multi-mode oscillator model has also been made, which would help to go beyond multi-mode oscillator approximation and to understand dissipative dynamics from another viewpoint. Since QD could be used to understand the efficiency of quantum gate operations in the absence of entanglement and quantum systems might suffer from quantum chaotic environment, our study should be helpful for not only further understanding quantum chaos, but also suppressing decoherence in quantum information processing.

This work is supported by National Natural Science Foundation of China under Grant No. 10774042.

[1] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997); E. Knill and R. Laflamme, Phys. Rev. Lett. **81**, 5672 (1998).
[2] J. Maziero *et al.*, Phys. Rev. A **80**, 044102 (2009).
[3] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001); M. Horodecki *et al.*, Phys. Rev. A **71**, 062307 (2005); S. Luo, Phys. Rev. A **77**, 042303 (2008); C. A. Rodriguez-Rosario *et al.*, J. Phys. A **41**, 205301 (2008);

R. Dillenschneider, Phys. Rev. B **78**, 224413 (2008); M. Piani *et al.*, Phys. Rev. Lett. **100**, 090502 (2008); M. S. Sarandy, Phys. Rev. A **80**, 022108 (2009); T. Werlang *et al.*, Phys. Rev. A **80**, 024103 (2009); R. Dillenschneider and E. Lutz, Europhys. Lett. **88**, 50003 (2009); A. Shabani and D. A. Lidar, Phys. Rev. Lett. **102**, 100402 (2009); M. Piani *et al.*, Phys. Rev. Lett.

- 102**, 250503 (2009); K. Modi, *arXiv:0902.0735*; A. Ferraro *et al.*, Phys. Rev. A **81**, 052318 (2010); J. Maziero *et al.*, Phys. Rev. A **81**, 022116 (2010).
- [4] F. F. Fanchini *et al.*, Phys. Rev. A **81**, 052107 (2010).
- [5] B. Wang *et al.*, Phys. Rev. A **81**, 014101 (2010).
- [6] R. P. Feynman and F. L. Vernon, Ann. Phys. **24**, 118 (1963); A. J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987).
- [7] H.-K. Park and S. W. Kim, Phys. Rev. A **67**, 060102(R) (2003); R. Blume-Kohout and W. H. Zurek, Phys. Rev. A **68**, 032104 (2003); J. W. Lee, D. V. Averin, G. Benenti, and D. L. Shepelyansky, Phys. Rev. A **72**, 012310 (2005); L. Ermann, J. P. Paz, and M. Saraceno, Phys. Rev. A **73**, 012302 (2006); C. Pineda, and T. H. Seligman, Phys. Rev. A **75**, 012106 (2007); C. Pineda, T. Gorin, and T. H. Seligman, New J. Phys. **9**, 106 (2007).
- [8] J. N. Bandyopadhyay, Europhys. Lett. **85**, 50006 (2009).
- [9] D. Rossini, G. Benenti, and G. Casati, Phys. Rev. E **74**, 036209 (2006).
- [10] F. Haake, J. Mod. Optic. **47**, 2883 (2000).
- [11] K. Modi *et al.*, Phys. Rev. Lett. **104**, 080501 (2010).
- [12] V. Vedral, Phys. Rev. Lett. **90**, 050401 (2003).
- [13] F. Haake *et al.*, Z. Phys. B - Condensed Matter **65**, 381 (1987).
- [14] W. M. Zhang *et al.*, Rev. Mod. Phys. **62**, 867 (1990).
- [15] A. Peres, Phys. Rev. A **30**, 1610 (1984); T. Gorina *et al.*, Phys. Rep. **435**, 33 (2006).
- [16] R. Sankaranarayanan and A. Lakshminarayan, Phys. Rev. E **68**, 036216 (2003).
- [17] A. Peres, Quantum Theory: Concepts and Methods, (Kluwer Academic Publishers, New York, Boston, Dordrecht, London, Moscow, 2002).
- [18] M. Ali *et al.*, Phys. Rev. A **81**, 042105 (2010).
- [19] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997).
- [20] L. Mazzola, J. Piilo, and S. Maniscalco, Phys. Rev. Lett. **104**, 200401 (2010); J.-S. Xu, *et al.*, Nat. Commun. **1**, 7 (2010).
- [21] B. Bellomo *et al.*, Phys. Rev. Lett. **99**, 160502 (2007).